8.2 Exercise Set

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Concept Reinforcement *Classify each of the following statements as either true or false.*

1. The quadratic formula can be used to solve *any* quadratic equation. True
2. The steps used to derive the quadratic formula are the same as those used when solving by completing the square. True
3. The quadratic formula does not work if solutions are imaginary numbers. False
4. Solving by factoring is always slower than using the quadratic formula. False
5. A quadratic equation can have as many as four solutions. False
6. It is possible for a quadratic equation to have no real-number solutions. True

Solve.

7. $2x^2 + 3x - 5 = 0$ $-\frac{5}{2}, 1$
8. $3x^2 - 7x + 2 = 0$ $\frac{1}{3}, 2$
9. $u^2 + 2u - 4 = 0$ $-1 \pm \sqrt{5}$
10. $u^2 - 2u - 2 = 0$ $1 \pm \sqrt{3}$
11. $3p^2 = 18p - 6$ $3 \pm \sqrt{7}$
12. $3u^2 = 8u - 5$ $1, \frac{5}{3}$
13. $h^2 + 4 = 6h$ $3 \pm \sqrt{5}$
14. $t^2 + 4t = 1$ $-2 \pm \sqrt{5}$
15. $x^2 = 3x + 5$ $\frac{3}{2} \pm \frac{\sqrt{29}}{2}$
16. $x^2 + 5x = -3$ $-\frac{5}{2} \pm \frac{\sqrt{13}}{2}$
17. $3t(t + 2) = 1$ $-1 \pm \frac{2\sqrt{3}}{3}$
18. $2t(t + 2) = 1$ $-1 \pm \frac{\sqrt{6}}{2}$
19. $\frac{1}{x^2} - 3 = \frac{8}{x}$ ⊡
20. $\frac{9}{x} - 2 = \frac{5}{x^2}$ ⊡
21. $t^2 + 10 = 6t$ $3 \pm i$
22. $t^2 + 10t + 26 = 0$ $-5 \pm i$
23. $x^2 + 4x + 6 = 0$ $-2 \pm \sqrt{2}i$
24. $x^2 + 11 = 6x$ $3 \pm \sqrt{2}i$
25. $12t^2 + 17t = 40$ $-\frac{8}{3}, \frac{5}{4}$
26. $15t^2 + 7t = 2$ $-\frac{2}{3}, \frac{1}{5}$
27. $25x^2 - 20x + 4 = 0$ $\frac{2}{5}$
28. $36x^2 + 84x + 49 = 0$ $-\frac{7}{6}$
29. $7x(x + 2) + 5 = 3x(x + 1)$ $-\frac{11}{8} \pm \frac{\sqrt{41}}{8}$
30. $5x(x - 1) - 7 = 4x(x - 2)$ $-\frac{3}{2} \pm \frac{\sqrt{37}}{2}$
31. $14(x - 4) - (x + 2) = (x + 2)(x - 4)$ 5, 10
32. $11(x - 2) + (x - 5) = (x + 2)(x - 6)$ 1, 15

⊡ Answers to Exercises 19 and 20 are on p. IA-17.

33. $5x^2 = 13x + 17$ $\frac{13}{10} \pm \frac{\sqrt{509}}{10}$
34. $25x = 3x^2 + 28$ $\frac{4}{3}, 7$
35. $x(x - 3) = x - 9$ $2 \pm \sqrt{5}i$
36. $x(x - 1) = 2x - 7$ $\frac{3}{2} \pm \frac{\sqrt{19}}{2}i$
37. $x^3 - 8 = 0$ (*Hint*: Factor the difference of cubes. Then use the quadratic formula.) $2, -1 \pm \sqrt{3}i$
38. $x^3 + 1 = 0$ $-1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
39. Let $g(x) = 4x^2 - 2x - 3$. Find x such that $g(x) = 0$. $\frac{1}{4} \pm \frac{\sqrt{13}}{4}$
40. Let $f(x) = 6x^2 - 7x - 20$. Find x such that $f(x) = 0$. $-\frac{4}{3}, \frac{5}{2}$
41. Let
$$g(x) = \frac{2}{x} + \frac{2}{x + 3}.$$
Find all x for which $g(x) = 1$. $-2, 3$
42. Let
$$f(x) = \frac{7}{x} + \frac{7}{x + 4}.$$
Find all x for which $f(x) = 1$. $5 \pm \sqrt{53}$
43. Let
$$F(x) = \frac{x + 3}{x} \text{ and } G(x) = \frac{x - 4}{3}.$$
Find all x for which $F(x) = G(x)$. $\frac{7}{2} \pm \frac{\sqrt{85}}{2}$
44. Let
$$f(x) = \frac{3 - x}{4} \text{ and } g(x) = \frac{1}{4x}.$$
Find all x for which $f(x) = g(x)$. $\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

Solve. Use a calculator to approximate, to three decimal places, the solutions as rational numbers.

45. $x^2 + 4x - 7 = 0$ −5.317, 1.317
46. $x^2 + 6x + 4 = 0$ −5.236, −0.764
47. $x^2 - 6x + 4 = 0$ 0.764, 5.236
48. $x^2 - 4x + 1 = 0$ 0.268, 3.732
49. $2x^2 - 3x - 7 = 0$ −1.266, 2.766
50. $3x^2 - 3x - 2 = 0$ −0.457, 1.457

TW 51. Are there any equations that can be solved by the quadratic formula but not by completing the square? Why or why not?

TW **52.** Suppose you are solving a quadratic equation with no constant term ($c = 0$). Would you use factoring or the quadratic formula to solve? Why?

SKILL REVIEW

To prepare for Section 8.3, review multiplying and simplifying radical expressions and complex-number expressions (Sections 7.3, 7.5, and 7.8).

Multiply and simplify.

53. $(x - 2i)(x + 2i)$ [7.8] $x^2 + 4$

54. $(x - 6\sqrt{5})(x + 6\sqrt{5})$ [7.5] $x^2 - 180$

55. $(x - (2 - \sqrt{7}))(x - (2 + \sqrt{7}))$ [7.5] $x^2 - 4x - 3$

56. $(x - (-3 + 5i))(x - (-3 - 5i))$ [7.8] $x^2 + 6x + 34$

Simplify.

57. $\dfrac{-6 \pm \sqrt{(-4)^2 - 4(2)(2)}}{2(2)}$ [7.3] $-\frac{3}{2}$

58. $\dfrac{-(-1) \pm \sqrt{(6)^2 - 4(3)(5)}}{2(3)}$ [7.8] $\frac{1}{6} \pm \frac{\sqrt{6}}{3}i$

SYNTHESIS

TW **59.** Explain how you could use the quadratic formula to help factor a quadratic polynomial.

TW **60.** If $a < 0$ and $ax^2 + bx + c = 0$, then $-a$ is positive and the equivalent equation, $-ax^2 - bx - c = 0$, can be solved using the quadratic formula.

a) Find this solution, replacing a, b, and c in the formula with $-a$, $-b$, and $-c$ from the equation.

b) How does the result of part (a) indicate that the quadratic formula "works" regardless of the sign of a?

For Exercises 61–63, let

$$f(x) = \frac{x^2}{x - 2} + 1 \quad \textit{and} \quad g(x) = \frac{4x - 2}{x - 2} + \frac{x + 4}{2}.$$

61. Find the x-intercepts of the graph of f. $(-2, 0), (1, 0)$

62. Find the x-intercepts of the graph of g. $(-5 - \sqrt{37}, 0), (-5 + \sqrt{37}, 0)$

63. Find all x for which $f(x) = g(x)$. $4 - 2\sqrt{2}, 4 + 2\sqrt{2}$

Solve. Approximate the solutions to three decimal places.

64. $x^2 - 0.75x - 0.5 = 0$ $-0.425, 1.175$

65. $z^2 + 0.84z - 0.4 = 0$ $-1.179, 0.339$

Solve.

66. $(1 + \sqrt{3})x^2 - (3 + 2\sqrt{3})x + 3 = 0$ $\sqrt{3}, \frac{3 - \sqrt{3}}{2}$

67. $\sqrt{2}x^2 + 5x + \sqrt{2} = 0$ $\frac{-5\sqrt{2}}{4} \pm \frac{\sqrt{34}}{4}$

68. $ix^2 - 2x + 1 = 0$ $-i \pm i\sqrt{1 - i}$

69. One solution of $kx^2 + 3x - k = 0$ is -2. Find the other. $\frac{1}{2}$

TW **70.** Can a graph be used to solve *any* quadratic equation? Why or why not?

TW **71.** Solve Example 2 graphically and compare with the algebraic solution. Which method is faster? Which method is more precise?

TW **72.** Solve Example 4 graphically and compare with the algebraic solution. Which method is faster? Which method is more precise?

Try Exercise Answers: Section 8.2

7. $-\frac{5}{2}, 1$ **35.** $2 \pm \sqrt{5}i$ **39.** $\frac{1}{4} \pm \frac{\sqrt{13}}{4}$ **43.** $\frac{7}{2} \pm \frac{\sqrt{85}}{2}$
45. $-5.317, 1.317$

8.3 Studying Solutions of Quadratic Equations

- The Discriminant
- Writing Equations from Solutions

THE DISCRIMINANT

It is sometimes enough to know what *type* of number a solution will be, without actually solving the equation. Suppose we want to know if $4x^2 + 7x - 15 = 0$ has rational solutions (and thus can be solved by factoring). Using the quadratic formula, we would have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4 \cdot 4 \cdot (-15)}}{2 \cdot 4}.$$